Computing Without Bits

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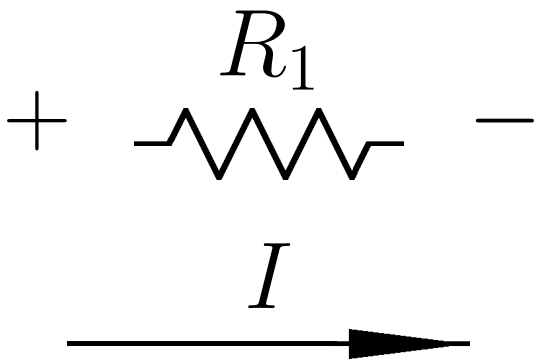
Analog Computing

- Voltage levels rather than numerals
- Continuous voltages, unlimited precision but limited accuracy
- Basic elements: voltage divider, adder, integrator
- Special devices: multipliers, function generators
- Solves differential equations

Resistors



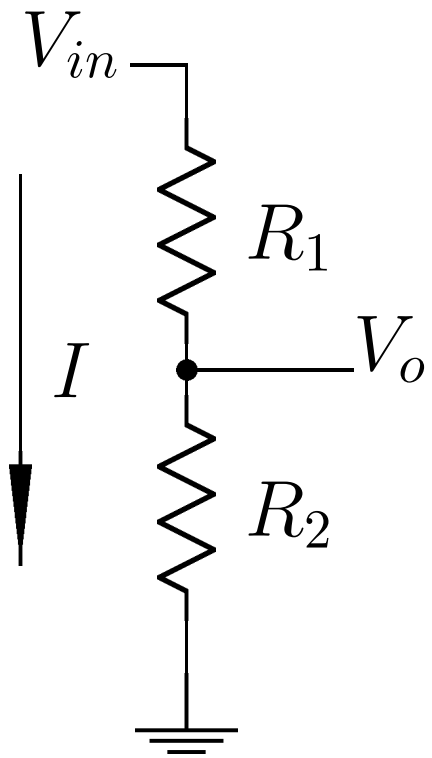
- Governed by Ohm's Law:

$$V = IR_1$$

- Add in series

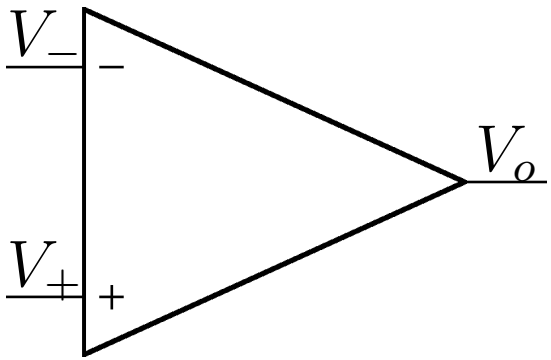
$$R_s = R_1 + R_2$$

Voltage Dividers/Multiplying by Constants Less Than 1



- $I = \frac{V_{in}}{R_1+R_2}$
- $V_o = IR_2$
- $V_o = V_{in}\frac{R_2}{R_1+R_2}$

Operational Amplifiers



- Approximate characteristics
  - Infinite gain

$$\frac{V_o}{V_+ - V_-} = \infty$$

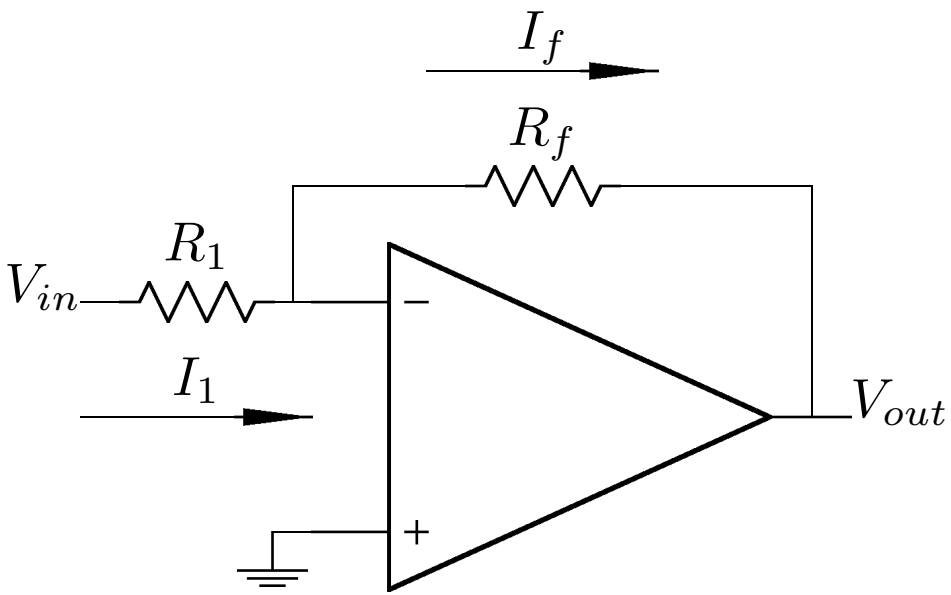
  - Infinite input resistance
- Implications
  - No differential input voltage

$$V_+ = V_-$$

  - No input current

$$I_+ = I_- = 0$$

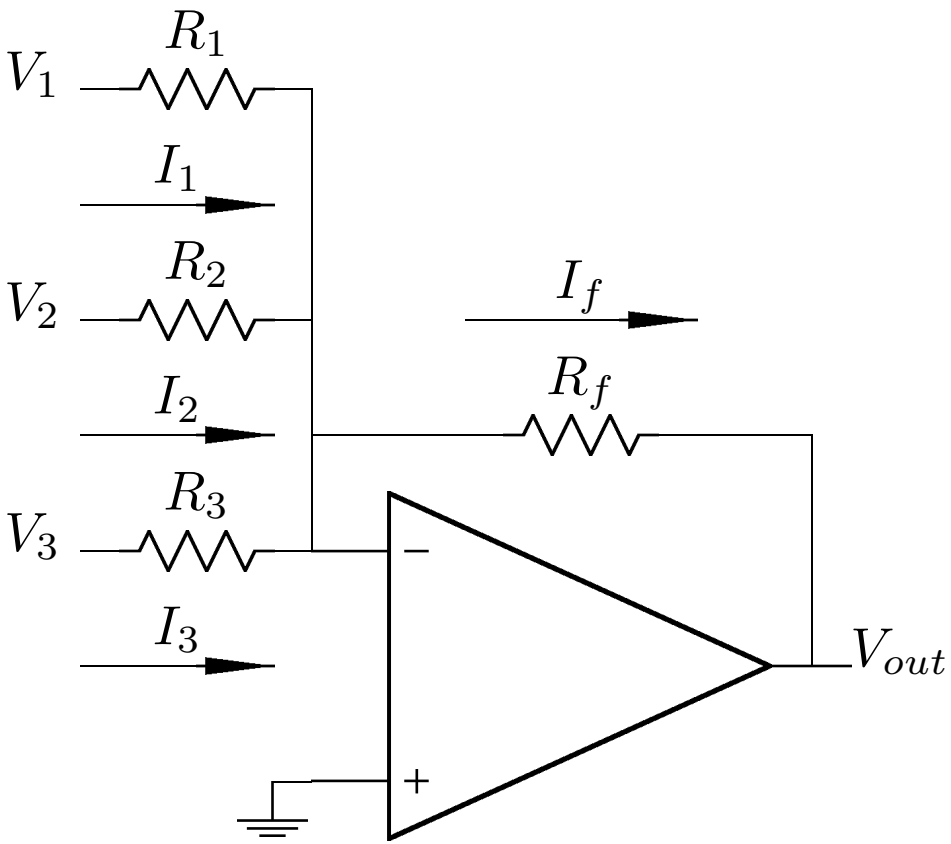
Inverting Amplifier/Multiplying by Any Constant



- V_- is at "virtual ground:" 0 Volts
- $I_1 = I_f$
- $I_1 = \frac{V_{in}}{R_1}$
- $I_f = -\frac{V_{out}}{R_f}$
- $V_{out} = -I_f R_f = -I_1 R_f$

$$V_{out} = -\frac{R_f}{R_1} V_{in}$$

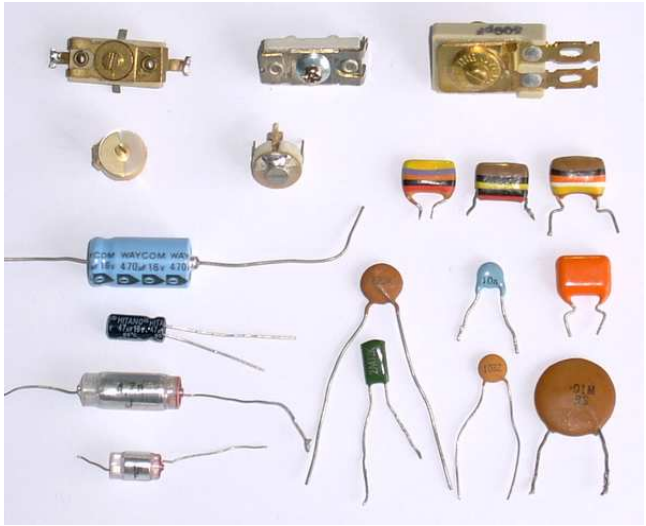
Weighted Adder

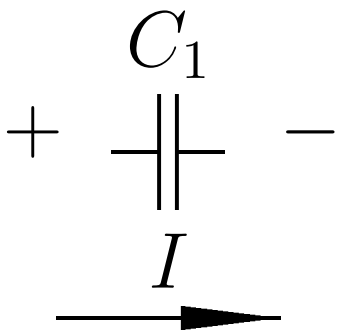


- $I_1 + I_2 + I_3 = I_f$
- $I_1 = \frac{V_1}{R_1}$
- $I_f = -\frac{V_{out}}{R_f}$
- $V_{out} = -I_f R_f = -(I_1 + I_2 + I_3)R_f$

$$V_{out} = -\left(\frac{R_f}{R_1}V_1 + \frac{R_f}{R_2}V_2 + \frac{R_f}{R_3}V_3\right)$$

Capacitors



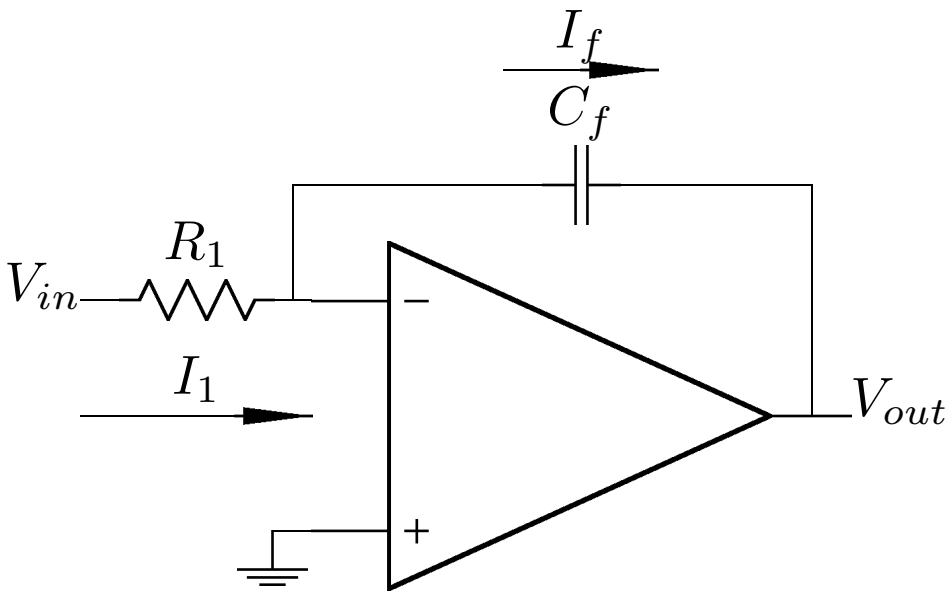


- Stores electrical charge
- Current proportional to change in voltage

$$I = C_1 \frac{dV}{dt}$$

$$V = \frac{1}{C_1} \int I dt$$

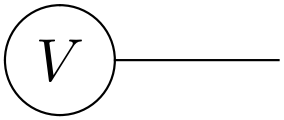
Integrator



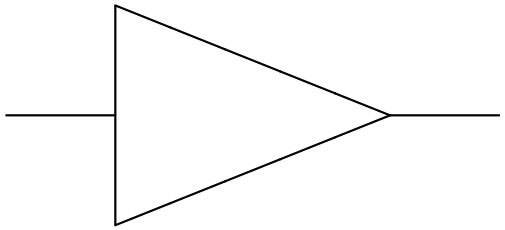
- $I_1 = I_f$
- $I_1 = \frac{V_{in}}{R_1}$
- $I_f = -C_f \frac{dV_{out}}{dt}$
- $V_{out} = -\frac{1}{C_f} \int I_f dt$
- $V_{out} = -\frac{1}{C_f} \int I_1 dt$

$$V_{out} = -\frac{1}{R_1 C_f} \int V_{in} dt$$

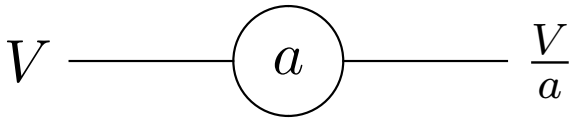
Analog Computer Symbols



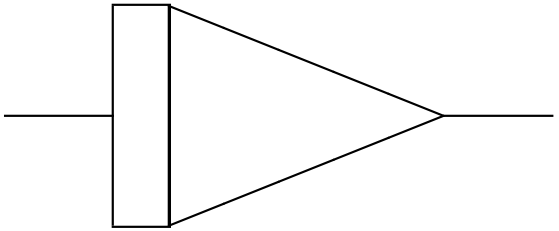
Constant Source



Weighted Adder



Voltage Divider

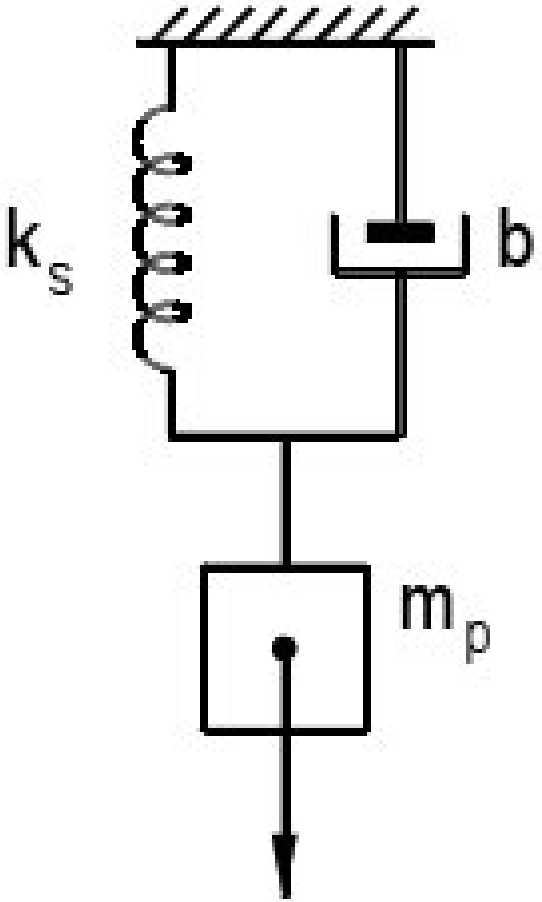


Integrator

Operating Modes

- Initial condition
- Operate/Run
- Hold
- Repeat
- Setup/Pot/Reset

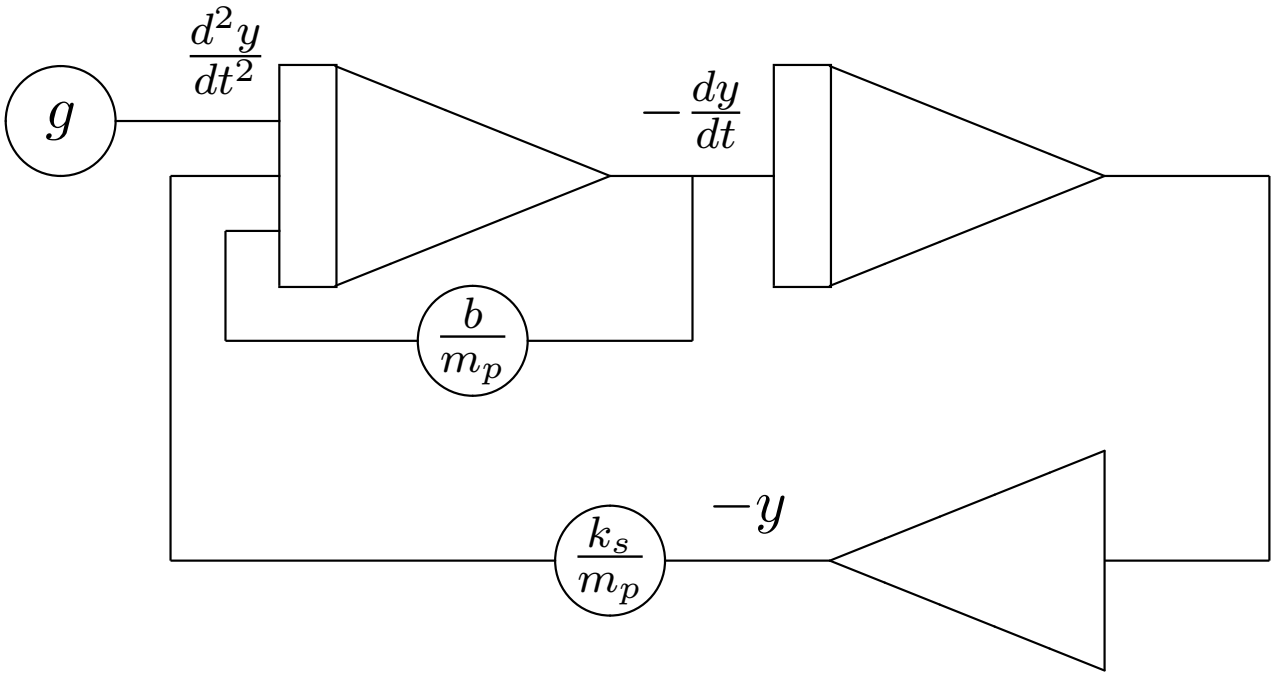
Example Problem: Damped Spring



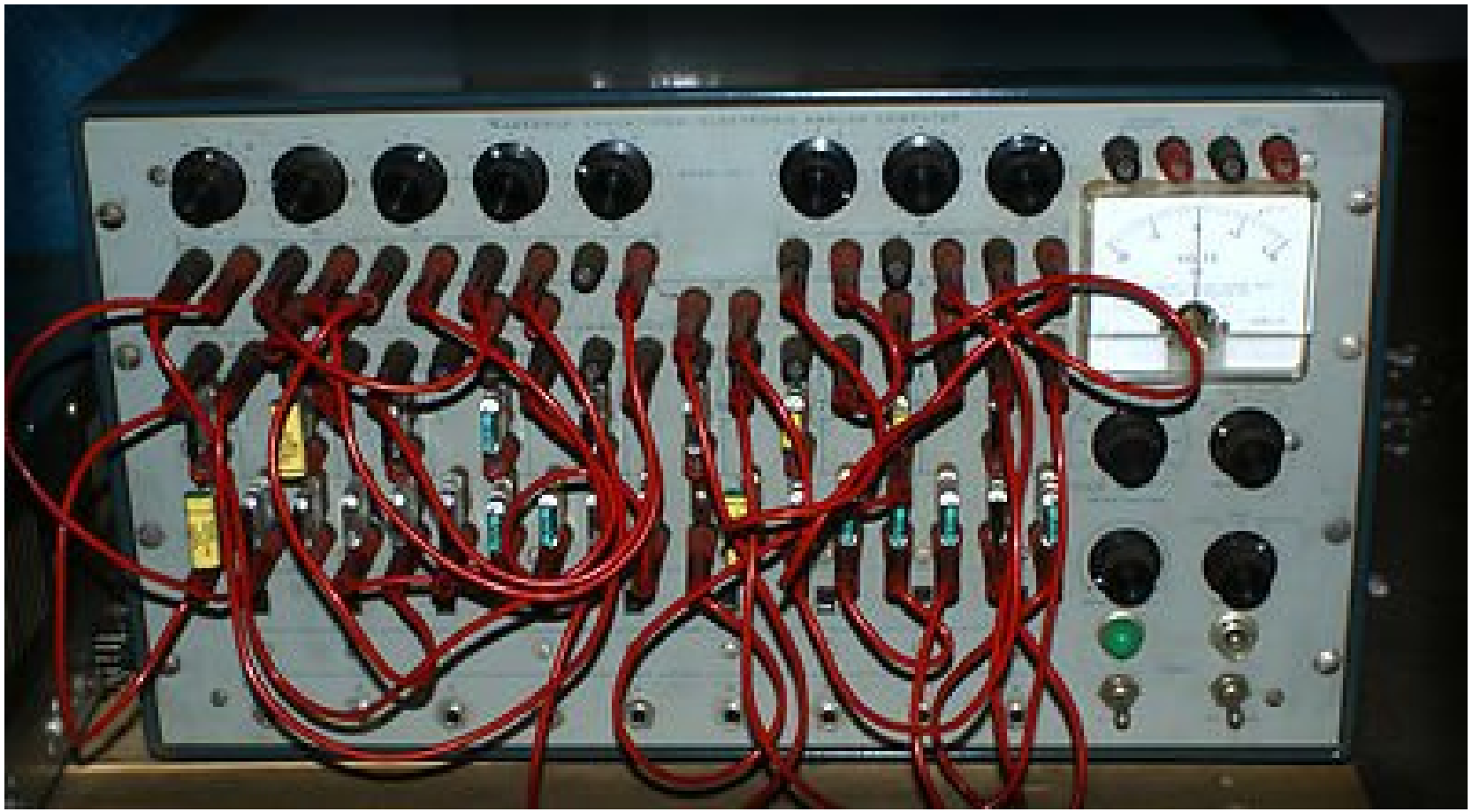
- $F = ma$
- Forces:
  - Gravity: $F_g = m_p g$
  - Spring tension: $F_s = -k_s y$
  - Damping: $F_d = -b\frac{dy}{dt}$
- Sum of forces: $F_g + F_s + F_d = m_p \frac{d^2y}{dt^2}$

$$\frac{d^2y}{dt^2} = -\frac{b}{m_p}\frac{dy}{dt} - \frac{k_s}{m_p}y + g$$

Spring Simulation



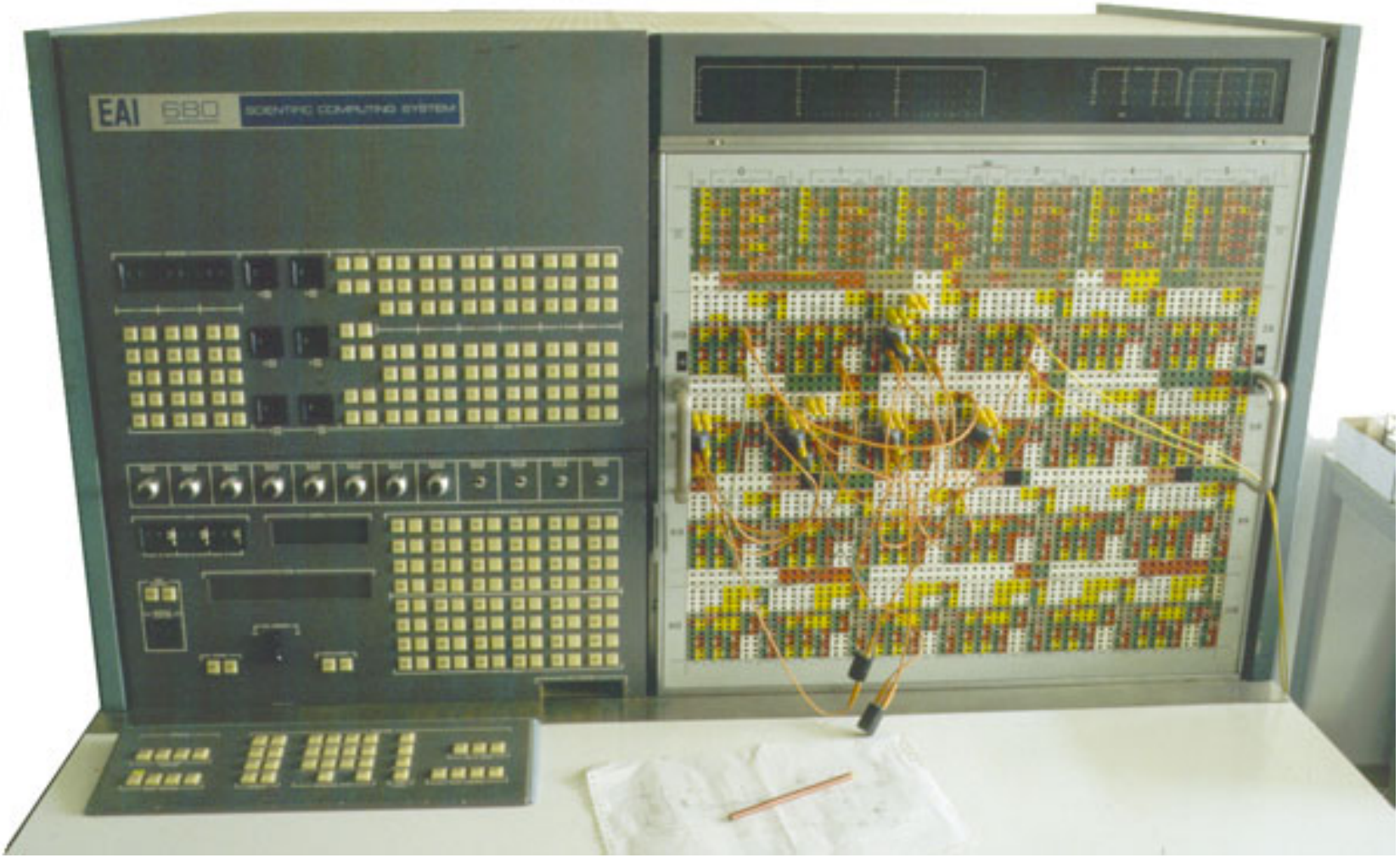
Example: Heathkit EC-1



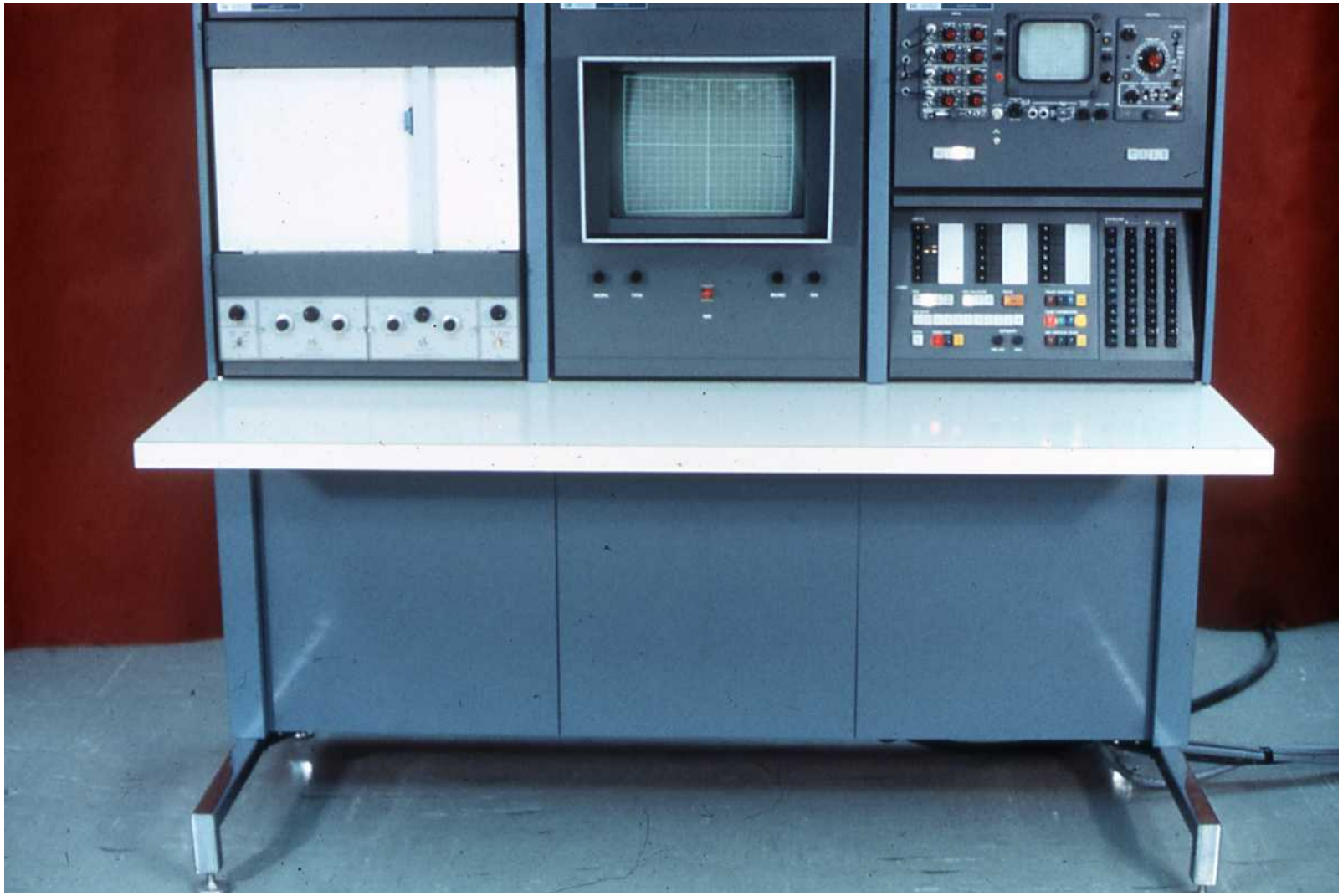
Example: Vogel

Example: AKAT

Example: EAI680



Example: EAI680 Output Devices



Hybrid Computers

- Integrating analog and digital computers
- Analog-to-digital converters
- Digital-to-analog converters
- Digital control of analog operate mode

Example: EAI690

Example: Large Hybrid Installation

Questions?